A LOOK AT NUMBERING SYSTEMS

As discussed in Chapter 2 of this text, a numbering system is a way of representing numbers. People generally use the *decimal numbering system* explained in Chapter 2 and reviewed next; computers process data using the *binary numbering system*. Another numbering system related to computer use is the *hexadecimal numbering system*, which can be used to represent long strings of binary numbers in a manner more understandable to people than the binary numbering system. Following a discussion of these three numbering systems, we take a look at conversions between numbering systems and principles of computer arithmetic, and then close with a look at how to perform conversions using a scientific calculator. ■

The Decimal and Binary Numbering System

The *decimal* (*base 10*) numbering system uses 10 symbols—the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9—to represent all possible numbers and is the numbering system people use most often. The *binary* (*base 2*) numbering system is used extensively by computers to represent numbers and other characters. This system uses only two digits—0 and 1. As illustrated in Figure 2-3 in Chapter 2, the place values (columns) in the binary numbering system are different from those used in the decimal system.

The Hexadecimal Numbering System

Computers often output diagnostic and memory-management messages and identify network adapters and other hardware in *hexadecimal* (*hex*) notation. Hexadecimal notation is a shorthand method for representing the binary digits stored in a computer. Because large binary numbers—for example, 11010100010011101—can easily be misread by people, hexadecimal notation groups binary digits into units of four, which, in turn, are represented by other symbols.

The hexadecimal numbering system is also called the *base 16 numbering system* because it uses 16 different symbols. Since there are only 10 possible numeric digits, hexadecimal uses letters instead of numbers for the additional 6 symbols. The 16 hexadecimal symbols and their decimal and binary counterparts are shown in Figure R-3.

Hexadecimal is not itself a code that the computer uses to perform computations or to communicate with other machines. This numbering system does, however, have a special relationship to the 8-bit bytes of ASCII and EBCDIC that makes it ideal for displaying addresses and other data quickly. As you can see in Figure R-3, each hex character has a 4-bit binary counterpart, so any combination of 8 bits can be represented by exactly two hexadecimal characters. For example, the letter N (represented in ASCII by 01001110) has a hex representation of *4E* (see the Binary Equivalent column in Figure R-3).

FIGURE R-3
Hexadecimal characters and their decimal and binary equivalents.

HEXADECIMAL CHARACTER	DECIMAL EQUIVALENT	BINARY EQUIVALENT
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Converting Between Numbering Systems

In Figure 2-3 in Chapter 2, we illustrated how to convert from binary to decimal. Three other types of conversions are discussed next.

Hexadecimal to Decimal

As shown in Figure R-4, the process for converting a hexadecimal number to its decimal equivalent is similar to converting a binary number to decimal, except the base number is 16 instead of 2. To determine the decimal equivalent of a hexadecimal number (such as 4F6A, as shown in Figure R-4), multiply the decimal equivalent of each individual hex character (determined by using the table in Figure R-3) by the appropriate power of 16 and then add the results to obtain the decimal equivalent of that hex number.

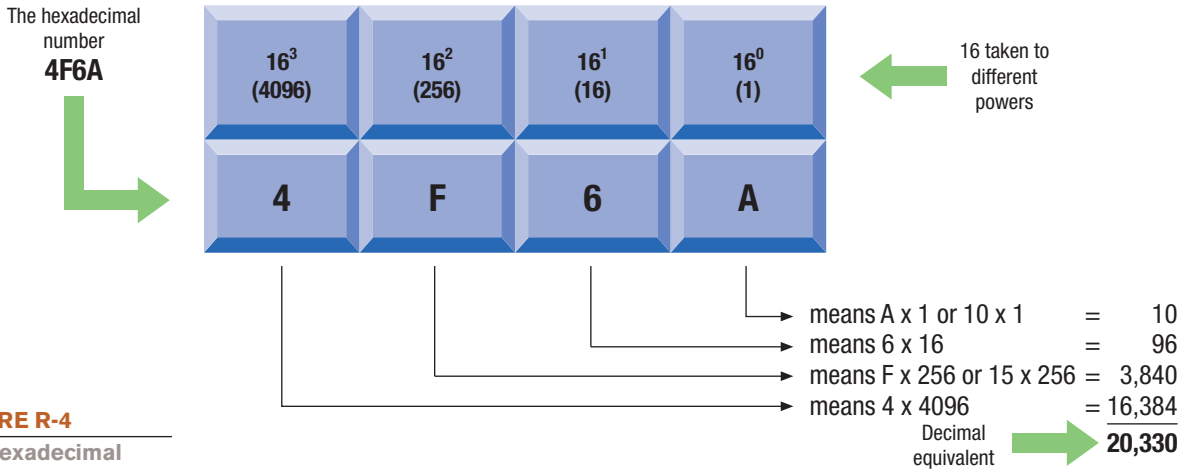


FIGURE R-4
The hexadecimal (base 16) numbering system. Each digit in a hexadecimal number represents 16 taken to a different power.

Hexadecimal to Binary and Binary to Hexadecimal

To convert from hexadecimal to binary, we convert each hexadecimal digit separately to 4 binary digits (using the table in Figure R-3). For example, to convert F6A9 to binary, we get

F	6	A	9
1111	0110	1010	1001

or 1111011010101001 in binary representation. To convert from binary to hexadecimal, we go through the reverse process. If the number of digits in the binary number is not divisible by 4, we add leading zeros to the binary number to force an even division. For example, to convert the binary number 1101101010011 to hexadecimal, we get

0001	1011	0101	0011
1	B	5	3

or lB53 in hexadecimal representation. Note that three leading zeros were added to change the initial 1 to 0001 before making the conversion.

Decimal to Binary and Decimal to Hexadecimal

To convert from decimal to either binary or hexadecimal, we can use the *remainder method*. To use the remainder method, the decimal number is divided by 2 (to convert to a binary number) or 16 (to convert to a hexadecimal number). The *remainder* of the division operation is recorded and the division process is repeated using the *quotient* as the next dividend, until the quotient becomes 0. At that point, the collective remainders (written backwards) represent the equivalent binary or hexadecimal number (see Figure R-5).

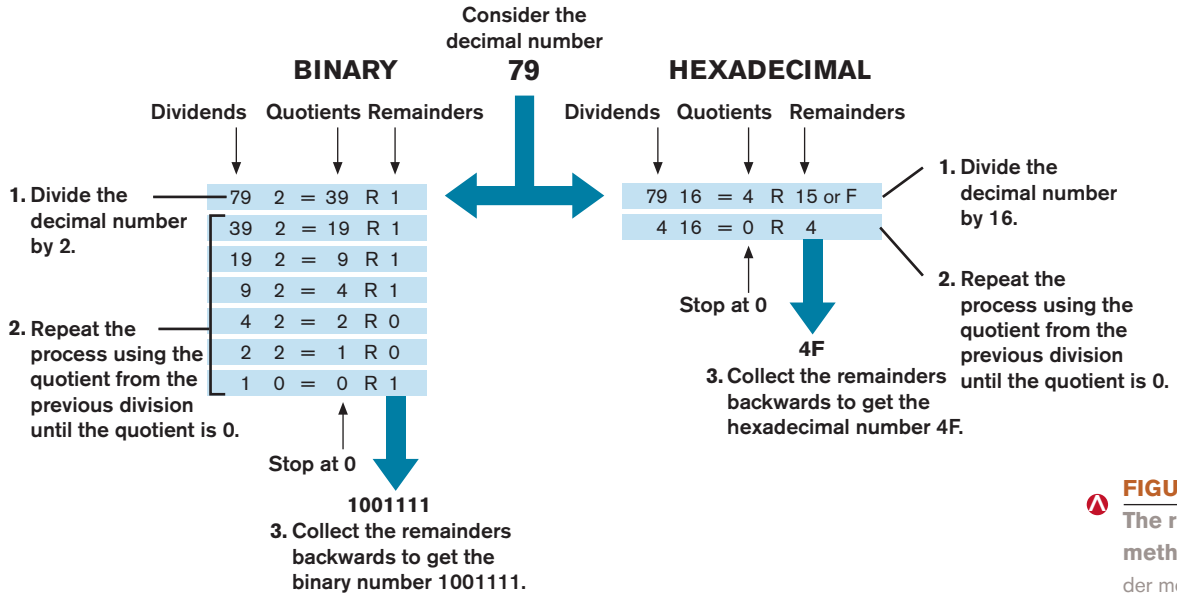


FIGURE R-5
The remainder method. The remainder method can be used to convert decimal numbers to binary or hex format.

A table summarizing all the numbering system conversion procedures covered in this text is provided in Figure R-6.

FROM BASE	TO BASE 2	TO BASE 10	TO BASE 16
2		Starting at the right-most digit, multiply binary digits by 2^0, 2^1, 2^2, etc., respectively and then add products.	Starting at the right-most digit, convert each group of four binary digits to a hex digit.
10	Divide repeatedly by 2 using each quotient as the next dividend until the quotient becomes 0, and then collect the remainders in reverse order.		Divide repeatedly by 16 using each quotient as the next dividend until the quotient becomes 0, and then collect the remainders in reverse order.
16	Convert each hex digit to four binary digits.	Starting at right-most digit, multiply hex digits by 16^0, 16^1, 16^2, etc., respectively. Then add products.	

FIGURE R-6
Summary of conversions.

Computer Arithmetic

To most people, decimal arithmetic is second nature. Addition and subtraction using binary and hexadecimal numbers is not much different than the same operations with decimal numbers—just the number of symbols used in each system varies. For instance, the digits in each column are added or subtracted and you carry to and borrow from the column to the left as needed as you move from right to left. Instead of carrying or borrowing 10, however—as you would in the decimal system—you carry or borrow 2 (binary) or 16 (hexadecimal).

Figure R-7 provides an example of addition and subtraction with decimal, binary, and hexadecimal numbers.

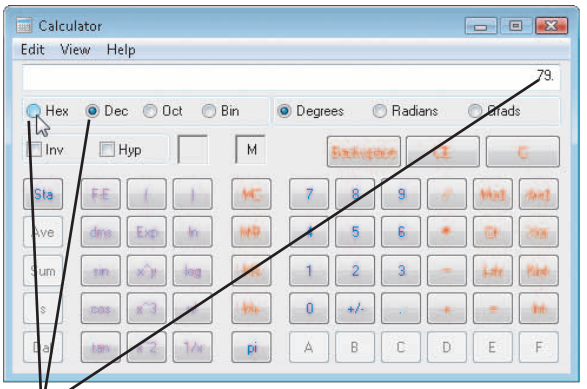
	DECIMAL	BINARY	HEXADECIMAL
Addition	$\overset{1}{144} + 27 = 171$	$\overset{111}{100101} + 10011 = 111000$	$\overset{1}{8E} + 2F = BD$
Subtraction	$\overset{3}{\cancel{14}}\overset{1}{4} - 27 = 117$	$\overset{0}{\cancel{1}}\overset{1}{\cancel{0}}\overset{0}{\cancel{0}}1\overset{1}{\cancel{0}}1 - 10011 = 10010$	$\overset{7}{\cancel{8}}\overset{1}{E} - 2F = 5F$

FIGURE R-7
Adding and subtracting with the decimal, binary, and hexadecimal numbering systems.

Using a Scientific Calculator

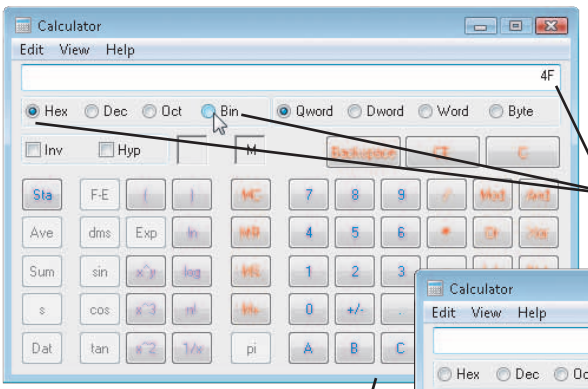
A scientific calculator can be used to convert numbers between numbering systems, or to check conversions performed by hand. Many conventional calculators have different numbering system options; scientific calculator programs can be used for this purpose, as well. For example, Figure R-8 shows how to use the Windows Calculator program to double-check the hand calculations performed in Figure R-5 (the *Scientific* option must be selected using the View menu to display the options shown in the figure). Arithmetic can also be performed in any numbering system on a calculator, once that numbering system is selected on the calculator. Notice that, depending on which numbering system is currently selected, not all numbers on the calculator are available—only the possible numbers are displayed, such as only 0 and 1 when the binary numbering system is selected.

FIGURE R-8
Using a scientific calculator. A physical calculator or calculator program can be used to convert between numbering systems, as well as to perform arithmetic in different numbering systems.

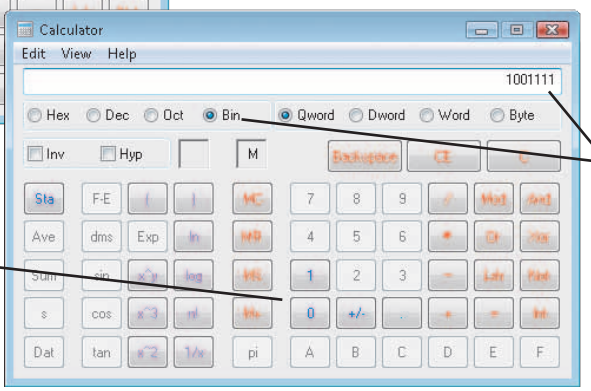


WINDOWS CALCULATOR
The Calculator program is typically located under Accessories on the Windows Start menu; select the *Scientific* option using the Calculator's View menu.

1. After entering a number (such as the decimal number 79 with the decimal numbering system selected shown here), select the numbering system to which the number should be converted (hex in this example).



2. The number is now displayed in hex notation. To convert it to binary, select that numbering system.



3. The number is now displayed in binary representation.

Numbers and operators can be used to perform arithmetic using the selected numbering system. Note that not all numbers and other symbols on the calculator are available—only the ones appropriate for the selected numbering system.